

## TABLE ERRATA

**591.**—G. W. SPENCELEY, R. M. SPENCELEY & E. R. EPPERSON, *Smithsonian Logarithmic Tables to Base e and Base 10*, The Smithsonian Institution, Washington, D. C., 1952; reprinted with corrections, 1960.

On page viii of the Introduction the following corrections are required in the factors used in the calculation of  $\ln \pi$ : the last five digits of the second, third, and fourth factors should read 72716, 72058, and 88337, respectively. The natural logarithm of the fourth factor should end in 882, and the value of  $\ln \pi$  should be correspondingly increased to end in 343 instead of 342.

On page ix, line 19, the last five digits of the 25D approximation to the natural logarithm of the fourth factor should read 88178 in place of 88052. The corresponding value of the common logarithm should end in 213 instead of 212 when rounded to 23D, and the resulting value of  $\log \pi$  should consequently end in 127. Thus, the emended values of  $\ln \pi$  and  $\log \pi$  are each correct to 23D.

On page x, line 17, the terminal digit of  $x^2/2$  should be 4 instead of 3.

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EDITORIAL NOTE: On page ix, line 22, a typographical error occurs: the last five digits of the 25D value of  $\log e$  (the modulus) should read 11289 in place of 11829. For notices of corrections in the main tables see *Math. Comp.*, v. 20, 1966, p. 471, MTE 396 and further references therein, also v. 17, 1963, p. 215, MTE 332.

**592.**—NATIONAL BUREAU OF STANDARDS, *Handbook of Mathematical Functions*, Appl. Math. Ser. No. 55 (M. Abramowitz & I. A. Stegun, eds.), U.S. Government Printing Office, Washington, D. C., 1964, pp. 346–347.

In testing the programs described in [1], several errors were detected in Table 8.3, Legendre Function—Second Kind  $Q_n(x)$ . Table 8.3 was completely recalculated and the following cases were found which exceed the stated maximum end-figure error (two units):

$Q_9(.99)$	is	−0.48875 677	and should be	−0.48875 680
$Q_{10}(.87)$	is	0.54659 757	and should be	0.54659 761
$Q_{10}(.90)$	is	0.41282 291	and should be	0.41282 288
$Q_{10}(.91)$	is	0.30602 901	and should be	0.30602 904
$Q_{10}(.94)$	is	−0.19666 273	and should be	−0.19666 280
$Q_{10}(.95)$	is	−0.40421 502	and should be	−0.40421 498
$Q_{10}(.96)$	is	−0.60564 435	and should be	−0.60564 426
$Q_{10}(.98)$	is	−0.81720 735	and should be	−0.81720 741
$Q_{10}(.99)$	is	−0.59305 105	and should be	−0.59305 100

No errors in excess of the stated maximum end-figure error were detected in Table 8.4, Derivative of the Legendre Function—Second Kind  $Q'_n(x)$ .

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I. F. W. J. OLVER & J. M. SMITH, "Legendre functions on the Cut," *J. Comput. Phys.* (Submitted.)

**593.**—A. E. CURZON, "Errors in 10-decimal tables of the roots of  $J_1(y)Y_1(\eta y) - J_1(\eta y)Y_1(\eta) = 0$ ".

The roots  $y_n(\eta)$  of  $J_1(y)Y_1(\eta y) - J_1(\eta y)Y_1(\eta) = 0$  have been calculated using a Newton iteration [1] involving the phase  $\theta_1$  and modulus  $M_1$  of the first order Bessel functions  $J_1$  and  $Y_1$ . No more than 3 iterations are required to obtain 10-decimal accuracy in  $y_n$ . The angle  $\theta_1(x)$  is obtained from the expression  $\theta_1(x) = \phi(x) + \pi I[\{x - \phi(x)\}/\pi]$ , where  $\phi(x) = \tan^{-1}\{Y_1(x)/J_1(x)\}$  and  $I[z]$  is the integral part of the floating point number  $z$ . The roots were calculated using standard NAG and IMSL Bessel function subroutines. It was found that previously published 10-decimal tables [2], [3] contain numerous errors. One table [2] gives  $y_n$  for  $\eta = 0.01(0.01)0.99$  and  $n = 1(1)5$ . Of these 495 results 368 differ by  $\geq 1 \times 10^{-10}$  from the newly calculated results, 239 differ by  $\geq 2 \times 10^{-10}$  and 12 differ by  $\geq 1 \times 10^{-9}$ . A second table [3] gives  $y_n$  for  $\eta = 0.001(0.001)0.300$  and  $n = 5(1)10$ . Of these 1800 results 1164 differ by  $\geq 1 \times 10^{-10}$  from the newly calculated results, 363 differ by  $\geq 2 \times 10^{-10}$ , 36 differ by  $\geq 5 \times 10^{-10}$  and none differs by  $\geq 1 \times 10^{-9}$ .

Table I lists the errors,  $\epsilon$ , as a function of  $\eta$  and  $n$  for the 12 cases having  $\epsilon \geq 1 \times 10^{-9}$ , where a given  $\epsilon$  is the previously calculated value of  $y_n$  [2] less the newly calculated value of  $y_n$ .

A table of  $y_n$  calculated to 10 decimals for  $\eta = 0.00(0.01)0.99$  and  $n = 1(1)10$  and a manuscript giving further information about the method of calculation and the errors associated with it have been deposited at the Depository of Unpublished Mathematical Tables.

TABLE I

List of errors  $\epsilon \geq 10^{-9}$  in previously published 10-decimal values of  $y_n$  [2] where  $\epsilon_n = (\text{previous value of } y_n) - (\text{newly calculated value of } y_n)$ .

$\eta$	$n$	$10^{10}\epsilon$
0.55	2	-12
0.64	1	-11
0.74	1	12
0.75	1	13
0.76	1	10
0.76	2	-10
0.77	1	-11
0.79	1	12
0.79	2	-13
0.99	1	15
0.97	1	10
0.99	1	10

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1. A. E. CURZON, "A method for solving  $J_1(x)Y_1(\rho x) - J_1(\rho x)Y_1(x) = 0$ ," *J. Comput. Phys.* (To appear.)
2. H. E. FETTIS & J. C. CASLIN, *An Extended Table of Zeros of Cross Products of Bessel Functions*, Report ARL 66-0023, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, 1966, pp. 1-29.
3. H. E. FETTIS & J. C. CASLIN, *More Zeros of Bessel Function Cross Products*, Report ARL 68-0209, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, 1968, pp. 15-21.